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RESEARCH MEMORANDUM

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EFFECT OF AERODYNAMIC HEATING ON THE FLUTTER OF A RECTANGULAR WING AT A MACH NUMBER OF 2

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SUMMARY

This paper is concerned with the flutter of a solid wing as affected by aerodynamic heating, which can cause a large momentary loss in torsional stiffness. Both experimental and analytical studies were conducted and good correlation between theory and experiment is shown.

The cantilever wing which was of solid aluminum-alloy construction, was tested "cold" at a Mach number of 2 and did not flutter, but was caused to flutter when tested in air preheated to 800° F at a Mach number of 2. A large transient loss in torsional stiffness due to aerodynamic heating resulted in a short period of flutter. Calculations by the use of the theory of Budiansky and Mayers (Journal of Aeronautical Sciences, December 1956) predicted the time at which the minimum stiffness would occur which was very close to the time at which the wing fluttered.

The aerodynamic theory used for the flutter analysis was the second-order theory of Van Dyke (NACA Report 1183). The experimental results are compared to a flutter calculation which included the computed loss in stiffness due to torsional heating.

INTRODUCTION

One of the major structural effects of aerodynamic heating on a solid wing is to cause a reduction of torsional stiffness. This loss of stiffness can be attributed to two causes: first, a change in material properties which reduces the modulus of rigidity, and second, a transient loss due to thermal stresses set up by a nonuniform chordwise temperature distribution which can occur in a highly accelerated flight. The reduction due to thermal stresses has been studied by Budiansky and Mayers (ref. 1) and they have shown that very large decreases in torsional stiffness may be encountered for aircraft being rapidly accelerated into high-speed flight.

Torsional stiffness is one of the primary flutter parameters. For some very simple cases, it can be shown that the flutter speed is directly proportional to the square root of the torsional rigidity. It is, therefore, obvious that the effect of aerodynamic heating on flutter may be important and even at times disastrous. The purpose of this paper is to present an experimental flutter result on a solid cantilever wing which was tested at a Mach number of 2 in air preheated to 800° F and to compare this experimental result with a calculation of the flutter speed and of the loss in torsional stiffness. The aerodynamic theory used for the flutter analysis was the second-order theory of Van Dyke (ref. 2).

SYMBOLS

A	chordwise cross-sectional area, sq ft
a_1, a_2, a_3	constants used in equation (3)
b	half chord, ft
c_p	specific heat of air, Btu/lb/°F
c_m	specific heat of wing material, Btu/lb/°F
E	modulus of elasticity, lb/ft^2
GJ	torsional stiffness, $lb\text{-}in.^2$
h_x	heat-transfer coefficient, Btu/(sq ft)(sec)(°F)
k	reduced frequency, $b\omega/V$
$\bar{k}$	conductivity of air, Btu/(sec)(sq ft)(°F/ft)
$L_1, L_2, L_3, L_4, M_1, M_2, M_3, M_4$	nonlinear aerodynamic coefficients (defined in eq. (1))
M	Mach number
M_w	first area moment about axis of twist, ft^3
N_{Pr}	Prandtl number, $c_p\mu/\bar{k}$
q	dynamic pressure, lb/sq ft

R_x	Reynolds number, $\rho Vx/\mu$
r	radial distance from axis of twist, ft
$t(x)$	wing thickness, ft
T	temperature at time τ, $^{\circ}R$
T_s	stagnation temperature, $^{\circ}R$
T_∞	free-stream static temperature, $^{\circ}R$
T_{aw}	adiabatic wall temperature, $^{\circ}R$
V	velocity, ft/sec
x_o	axis of rotation measured from leading edge based on chord, positive rearward
x,y,z	Cartesian coordinates
α	coefficient of thermal expansion, $1/^{\circ}F$
$\beta = \sqrt{M^2 - 1}$	
γ	ratio of specific heat
η_r	recovery factor
μ	viscosity, lb-sec/sq ft
ρ	air density, slugs/cu ft
ρ_m	density of wing material, lb/ft^3
σ_y	axial stress in span direction, lb/sq in.
τ	time, sec
λ	time parameter
ω_α	first torsional angular frequency, radians/sec

Subscripts:

eff	effective
i	initial

DESCRIPTION OF MODEL AND TESTS

Model

The model was constructed of aluminum alloy and had a rectangular plan form with a chord of 8 inches and span of $11\frac{3}{4}$ inches. The wing had a solid cross section which tapered from a 65A003 airfoil section at the tip to a 65A004 at the root. The model was swept back 10° as shown in figure 1 in order to raise the divergence speed above the maximum operating speed of the tunnel. The model was tested backwards, that is with the trailing edge of the 65A series airfoil acting as the leading edge. This was done so that the center of gravity would have a rearward location and thus lower the flutter speed so that it would fall within the operating limits of the tunnel. The instrumentation on the model consisted of two sets of strain gages near the root which were used to measure the bending and torsional frequency.

The model properties are given in the following table:

Aspect ratio of panel	1.468
Elastic-axis location, percent chord	62.5
Center-of-gravity location, percent chord	57.8
First bending frequency, cps	65
First torsion frequency, cps	246
Second bending frequency, cps	362
Nondimensional radius of gyration (squared), based on half chord	0.22029

The wing mass per unit length of span varies linearly from 0.067 slug/ft at the root to 0.0545 slug/ft at the tip.

Wind Tunnel

The 27- by 27-inch test section of the preflight jet of the Langley Pilotless Aircraft Research Station at Wallops Island, Va., was used for the test. This tunnel is a blowdown type which exhausts directly to the atmosphere. The air could be preheated to approximately 800° F at $M = 2$. The test section and model are shown in figure 2.

Test Results

Two tests were made; the first was conducted with "cold" air. The second test was made with the air preheated to the maximum temperature condition. The wing did not flutter for the cold run. For the hot test, the wing began to flutter after being exposed to the airstream for 2 seconds and continued to flutter for more than 2 seconds and then stopped. This phenomenon will be explained in a later section. The total test time was approximately 10 seconds.

The test conditions and flutter results are given in the following table:

Test	Stagnation temperature, °F	Test-section density, slugs/cu ft	Test-section air velocity, ft/sec	Flutter frequency, cps	q, lb/sq ft
Cold	325	0.00287	2020	-----	5855.37
Hot	800	.00204	2600	108.6	6895.2

ANALYSIS

This section is concerned with a presentation of the method of flutter calculations and of the method of calculating the loss in torsional stiffness due to aerodynamic heating.

Method of Flutter Calculations

The flutter calculations were made using the conventional Rayleigh-Ritz type of flutter analysis. Three degrees of freedom were used, namely, the uncoupled first bending, second bending, and first torsion. The usual flutter determinant as given, for example, in reference 3 was used.

However, instead of employing the more conventional linear unsteady aerodynamic theory (ref. 4) in the flutter analysis, the second-order theory of Van Dyke (ref. 5) was used. This theory takes into account the nonlinear effects of airfoil shape and thickness. It has been found that, for supersonic speeds, the location of the center of pressure is highly dependent on the airfoil shape and, since the location of center of pressure with respect to, say, the center of gravity may have very large effects on the flutter speed, it was decided to use the more exact

nonlinear theory. Since the reduced frequency of the test was small ($k = 0.087$), only first-order terms in frequency were included in the nonlinear analysis; however, a check was made for one frequency ratio which included third-order frequency terms and no appreciable effect was found. The nonlinear aerodynamic coefficients as derived from reference 4 are as follows:

$$L_1 = 0 \qquad L_2 = \frac{1}{\beta k} \qquad L_3 = \frac{1}{\beta k^2} \qquad L_4 = \frac{1}{\beta k}\left(\frac{\beta^2 - 1}{\beta^2} - 2x_o\right) - \frac{\gamma + 1}{k}\left(\frac{M}{\beta}\right)^6 \frac{A}{8b^2} + \frac{(6M^2 - 4)}{\beta^4 k} \frac{A}{8b^2}$$

$$M_1 = 0 \qquad M_2 = \frac{1}{\beta k}(1 - 2x_o) - \frac{\gamma + 1}{k}\left(\frac{M}{\beta}\right)^4 \frac{A}{8b^2} + \frac{4}{\beta^2 k} \frac{A}{8b^2} \qquad M_3 = \frac{1}{\beta k^2}(1 - 2x_o) - \frac{\gamma + 1}{k^2}\left(\frac{M}{\beta}\right)^4 + \frac{4}{\beta^2 k^2} \frac{A}{8b^2} \qquad (1)$$

$$M_4 = \frac{1}{\beta k}\left[\frac{4}{3}\left(\frac{\beta^2 - 1}{\beta^2}\right) - 2x_o\left(\frac{2\beta^2 - 1}{\beta^2}\right) + 4x_o^2\right] - \frac{\gamma + 1}{k}\left(\frac{M}{\beta}\right)^4 \frac{M_w}{4b^3} + \frac{4}{\beta^2 k} \frac{M_w}{4b^3} + \frac{4M^2}{\beta^4 k}(N - 1)\frac{Ax_o}{8b^2}$$

where

$$N = \frac{(\gamma + 1)}{2} \frac{M^2}{\beta^2}$$

Calculation of Loss of Torsional Stiffness Due to Aerodynamic Heating

The basic theory used in calculating loss in torsional stiffness has been intuitively derived in reference 1. Basically, the assumption made is that an axial stress σ_y "follows the fiber" so that in a twisted condition, a component of σ_y can act in such a direction as to introduce a twisting moment on the wing. The formula for calculating the effect is given in reference 1 and may be written as

$$GJ_{eff} = GJ_i + \int_A \sigma_y r^2 dA \qquad (2)$$

where σ_y is the axial stress of an element dA which is located r distance from the axis of twist, and the integration is performed over the chordwise cross section of the wing. Negative values of σ_y indicate compression and positive values indicate tension. For solid wings, such as the one tested, which have most of the mass located near the midchord, the center portion will not heat up as quickly as the edges. The cooler center portion tends to restrain the edges from expanding and, thus, causes compressive stresses in the edges which can reduce the

effective torsional stiffness. The problem then is to compute the values of σ_y which are caused by nonuniform heating of the wing.

The stress σ_y at a point x of an airfoil due to a change in temperature is

$$\sigma_y = E(a_1 + a_2 x + a_3 z) - E\alpha(T - T_i) \tag{3}$$

where a_1, a_2, and a_3 are constants to be determined by boundary conditions, α is the coefficient of thermal expansion, T is the temperature at point x at time τ, and T_i is the initial temperature. This formula is based on the assumption that plane sections remain plane during the deformation. Of course, this assumption is not valid at the tip, where the stress must reduce to zero. However, Budiansky and Mayers (ref. 1) have investigated this tip effect for a free-free beam having a double-wedge section. They show that for the aspect ratio of the present wing the change in frequency square is only of the order of 3 percent. It is thus evident that the neglect of the tip effect will not materially affect the results of this paper.

The conditions needed for determining the constants are that the integral of the stress σ_y over the cross-sectional area must be zero and that the integral of the first moment of the stress about the axis of twist must be zero as follows:

$$\left.\begin{aligned} \int_A \sigma_y \, dA &= 0 \\ \int_A x\sigma_y \, dA &= 0 \\ \int_A z\sigma_y \, dA &= 0 \end{aligned}\right\} \tag{4}$$

For a doubly symmetrical airfoil like a symmetrical wedge, both a_2 and a_3 are zero. For an airfoil having symmetry about one axis, say the x axis, then $a_3 = 0$. For the case described herein, the 65A004 airfoil is symmetrical about the x axis but not about the z axis; therefore, a_1 and a_2 must be calculated.

Temperature Calculations

The temperature distribution was calculated from the following formula:

$$T - T_i = (T_{aw} - T_i)\left(1 - e^{-\frac{\lambda}{t(x)}}\right) \tag{5}$$

where

$$\lambda = \frac{2\tau h_x}{\rho_m c_m}$$

and

$$T_{aw} = T_\infty\left(1 + \eta_r \frac{\gamma - 1}{2} M^2\right)$$

This formula is based on the assumption of one-dimensional heat flow which implies that there is no chordwise heat flow and that there is no temperature gradient normal to the wing surface.

The temperature distribution is a function of the heat-transfer coefficient h_x. Because of the relatively rough surface of the airfoil in the heat test, it is presumed that the flow across the wing was almost entirely turbulent. Therefore, the following turbulent heat-transfer formula was used:

$$h_x = \frac{0.029\bar{k}}{x}(R_x)^{0.8} N_{Pr}^{1/3} \tag{6}$$

where x is the distance from the leading edge, R_x is the Reynolds number based on x, and N_{Pr} is the Prandtl number.

Application to a Specific Example

The foregoing analysis for calculating the change in torsional frequency has been applied to the present wing. Since no closed analytical solution is available for the 65 series airfoil, it was necessary to perform the integrations by numerical means. This was accomplished by dividing the wing cross section into 18 stations, which were 1/20 of the chord, and 4 additional stations at the leading edge and trailing edge, which were 1/40 of the chord.

The heat-transfer coefficient was calculated for each section by using equation (6). The temperature distribution $T - T_i$ was computed

from equation (5). The temperature distribution across the chord at $\tau = 2$ seconds for the tip, midspan, and root is shown in figure 3. Note the large change in temperature indicated between the leading edge and the 0.6-chord position.

The following values of the various constants were used in this calculation:

V, ft/sec	2,600	T_s, °R	1,260
μ, lb-sec/sq ft . .	7×10^{-7}	T_o, °R	530
ρ, slugs/cu ft . .	0.00204	η_r	0.9
ρ_m, lb/cu ft . . .	168	$\bar{k}$, Btu/(sec)(sq ft)(°F/ft) . .	9.21×10^{-6}
c_m, Btu/lb/°F . . .	0.21	N_{Pr}	0.596
M	2		

The loss in torsional stiffness was computed by using equation (2) at three spanwise stations - the root, the midspan, and the tip. In these calculations the variation of the modulus of elasticity E with temperature was taken into account. A plot of the torsional stiffness is given in figure 4, where the ratio of the effective stiffness at time τ to the value at $\tau = 0$ is plotted against time. Note that the thinnest section, the tip, has suffered a greater loss in stiffness than the thicker sections. Since the condition of zero stress at the tip was not satisfied, the present calculation overestimates the loss in stiffness at the tip; however, it is felt that the tip effect will be relatively small and that it can be neglected for the present case.

With the value of the stiffness computed, the torsional frequency and modal shapes were computed by using the iteration procedure of reference 5. The bending stiffness was also computed by the use of the procedure of reference 5; however, the value of the bending stiffness used was calculated at each span station by taking into account the variation of E with temperature.

DISCUSSION OF RESULTS

The results of applying the method of flutter calculations to the present configuration and the calculated operational curve due to aerodynamic heating are shown in figure 5. The velocity coefficient $V/b\omega_\alpha$ is plotted against the frequency ratio $\omega_{h_1}/\omega_\alpha$. The flutter boundary is rather flat for most of the range of frequency ratio but turns up rapidly as a frequency ratio of unity is approached. The unstable region is above

the flutter curve. The calculated operational curve is also shown. The numbers shown along the curves indicate the time in seconds. At the beginning of the test the wing is in an unstressed condition. The value of the flutter speed coefficient $V/b\omega_\alpha$ is 5.05, the frequency ratio $\omega_{h_1}/\omega_\alpha$ is 0.262 and is plotted at $\tau = 0$ in figure 5. As the wing is nonuniformly heated by the airstream, the torsional and bending frequencies are changed. The torsion frequency is initially reduced due to the stresses resulting from the uneven aerodynamic heating and also to the reduction in modulus of elasticity. The maximum change in torsional frequency occurred at 2 seconds and it had a value of 152 cps or a 38-percent change in frequency. The bending frequency at 2 seconds was calculated to be 62.8 or a 3.4-percent change from the initial frequency. The flutter speed coefficient $V/b\omega_\alpha$ is plotted in figure 5 at the various times as indicated up to $\tau = 4$ seconds. This operational curve intersects the flutter region as indicated at about $\tau = 1$ second, and the wing remains in the unstable flutter region for about 4 additional seconds at which time, the wing, even though hotter, is more evenly heated and has regained some of its stiffness. In the experiment, the wing started to flutter at about 2 seconds and continued fluttering for slightly over 2 more seconds before stabilizing as indicated in the figure. Thus the calculations are in fairly good agreement with the incidence of flutter.

CONCLUDING REMARKS

This paper has been concerned with the effect of transient aerodynamic heating on the flutter of a solid aluminum-alloy wing. A model wing which did not flutter at a Mach number of 2 in air preheated to 300° F was caused to flutter at a Mach number of 2 when the air was preheated to a stagnation temperature of 800° F. The flutter is explained by the loss of torsional stiffness due to the thermal stresses set up as a result of the uneven aerodynamic heating. The flutter speed was calculated by using a nonlinear aerodynamic theory based on second-order theory of Van Dyke (NACA Report 1183). The loss of stiffness due to aerodynamic heating was calculated and the operational line intersected the flutter curve.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., March 19, 1958.

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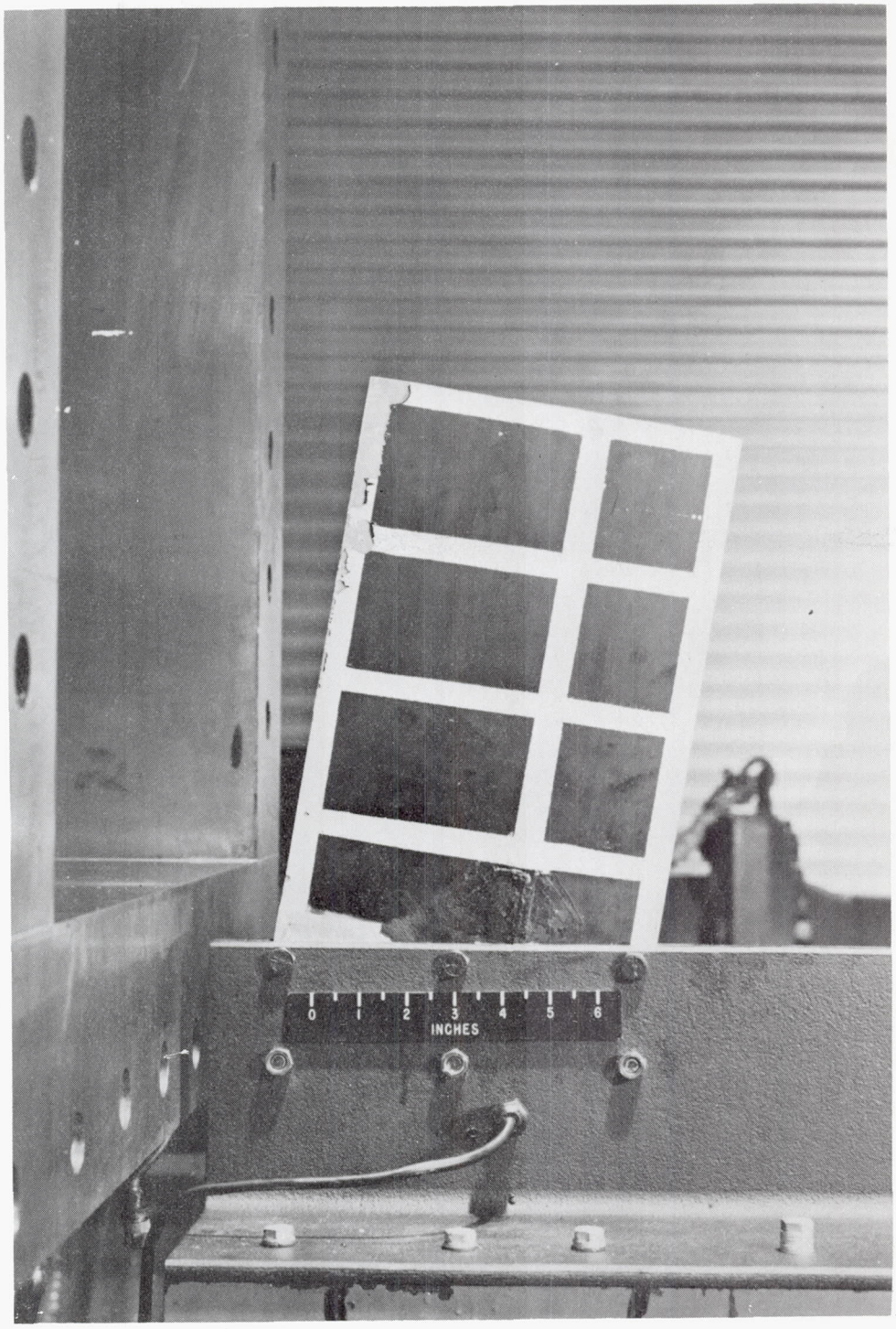


Figure 1.- View of model mounted in test section. L-91610

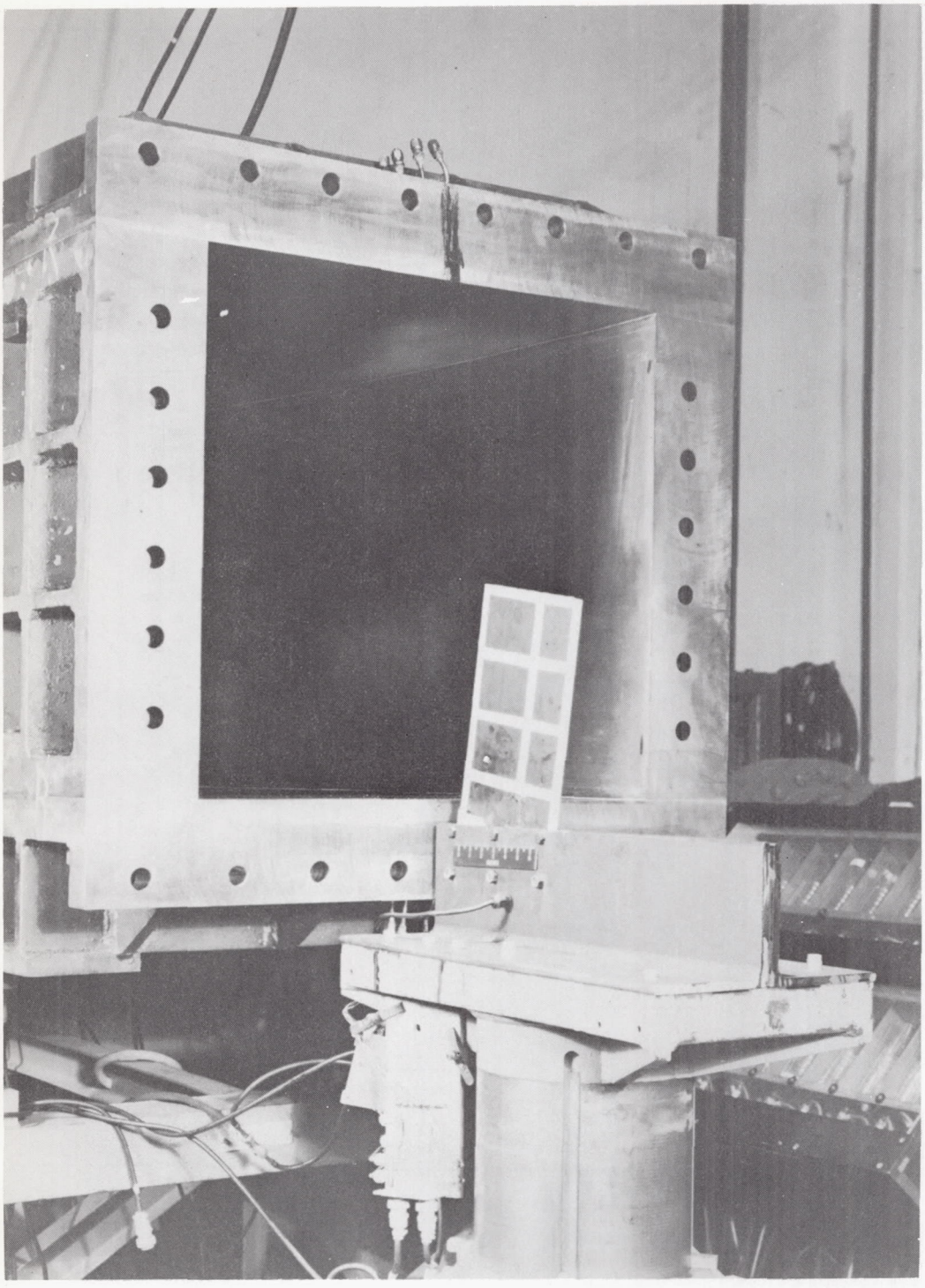


Figure 2.- View of model. L-91609

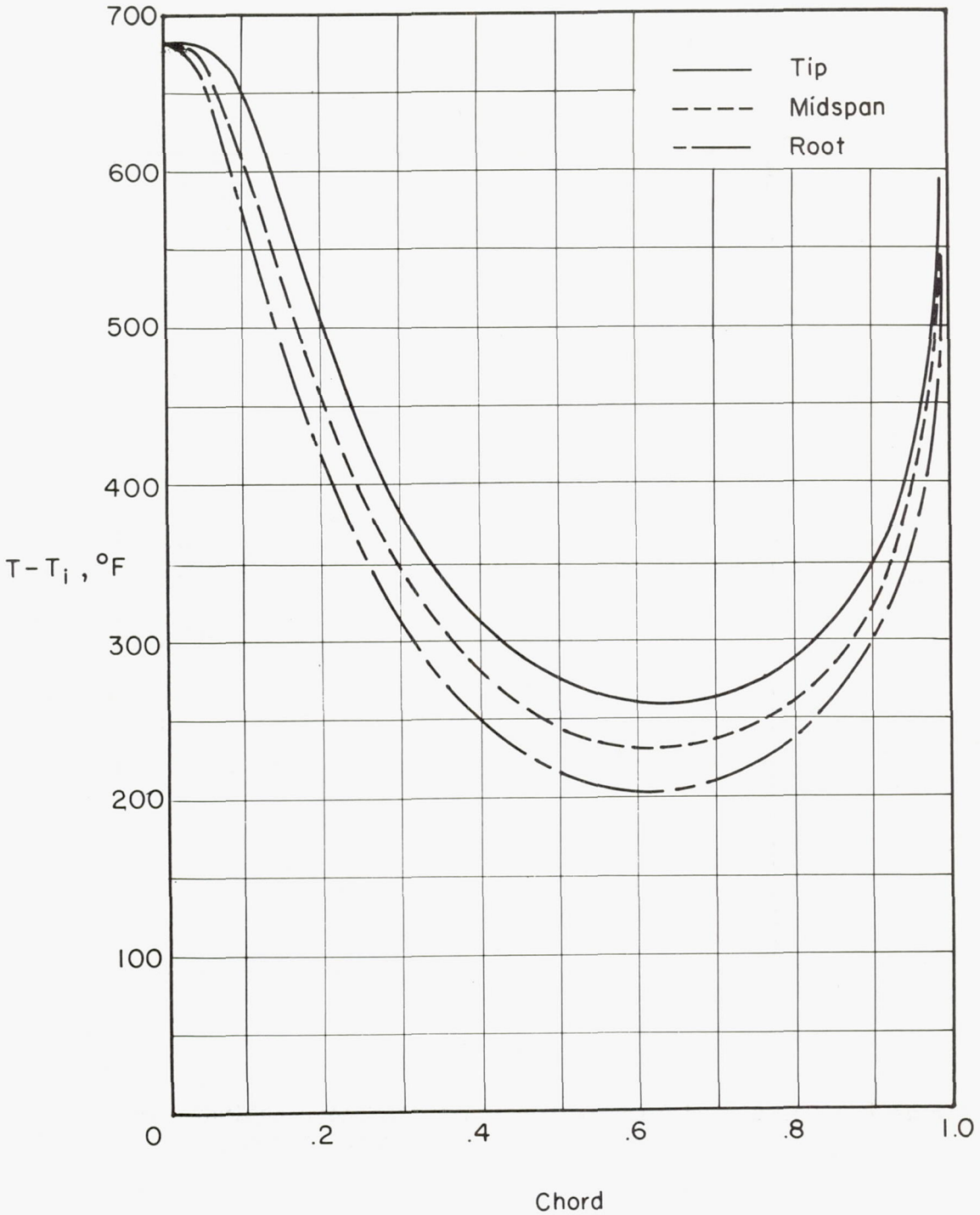


Figure 3.- Calculated chordwise temperature distribution at τ = 2 seconds, and M = 2.0.

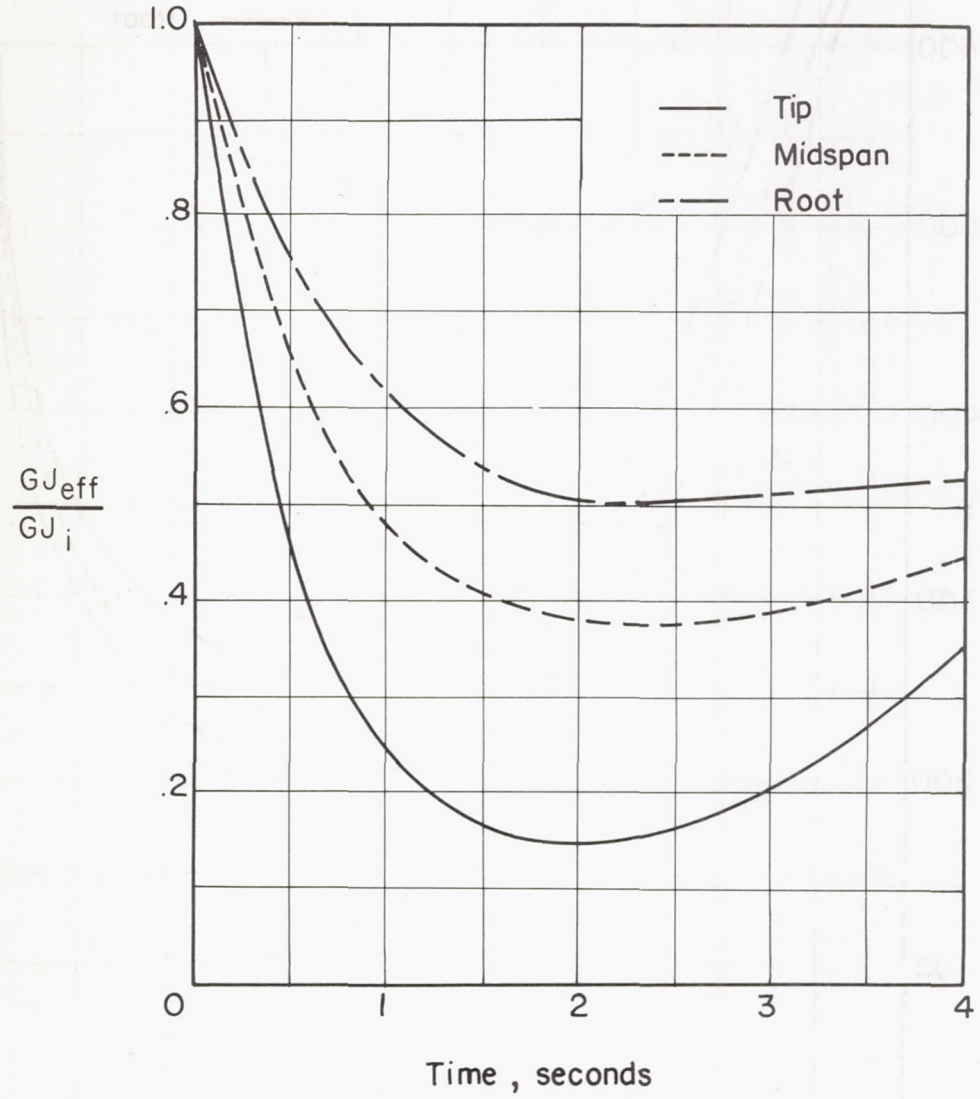


Figure 4.- Calculated loss in torsional stiffness against time at M = 2.0. (Tip uncorrected for zero stress.)

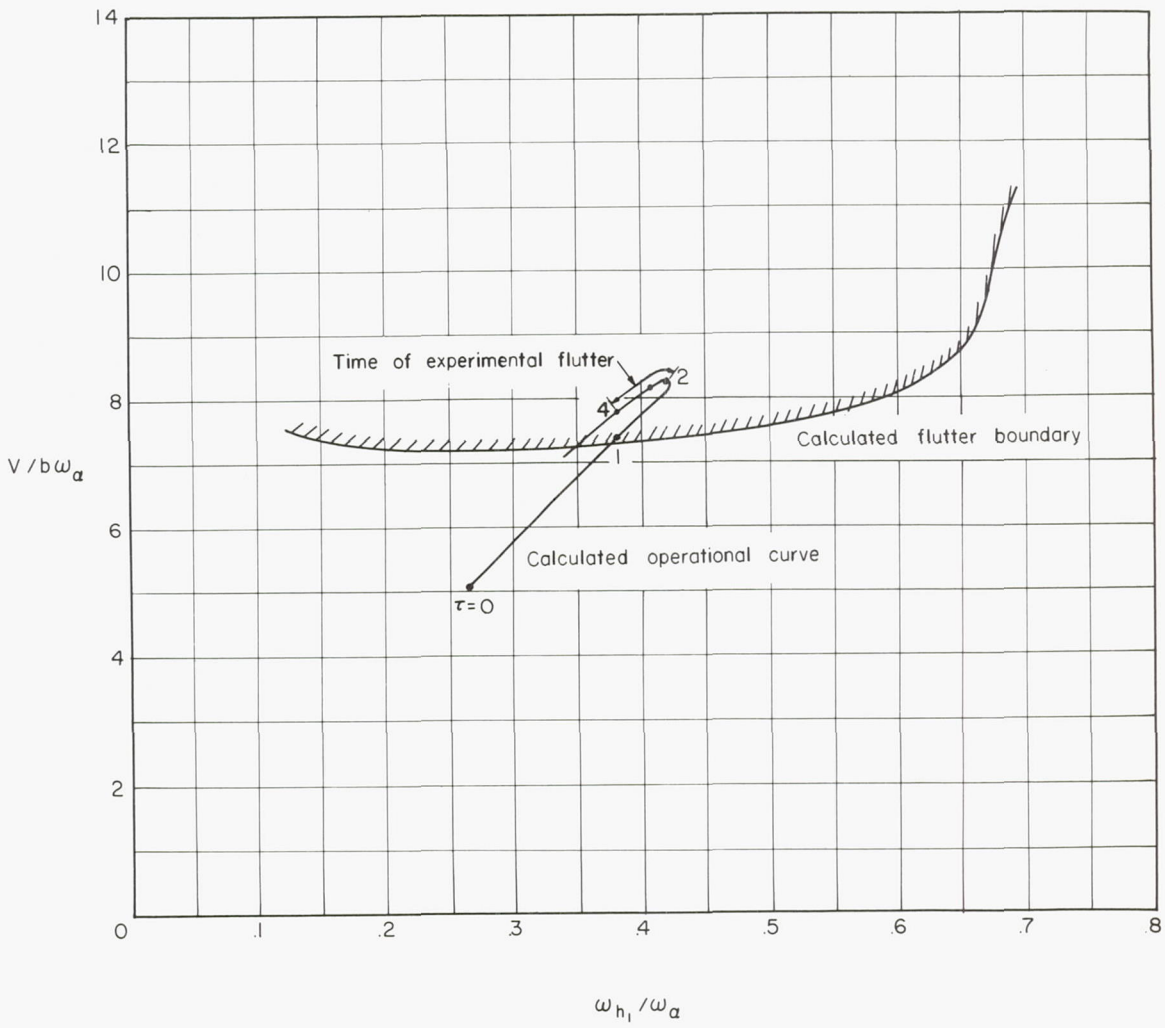


Figure 5.- Effect of aerodynamic heating on flutter.

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